what is the equation for rayleigh backscatter by radar as a function of size and complex refractive index?

The backscattering cross-section for a spherical particle in the Rayleigh scattering regime ($D \ll \lambda$) is given by:

$$\sigma_b = \frac{\pi^5 D^6}{\lambda^4} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2$$

Where:

- σ_b: Radar backscattering cross-section
- D: Particle diameter
- λ: Radar wavelength
- m: Complex refractive index of the particle ($m = n - ik$)

The term containing the refractive index is the squared magnitude of the complex dielectric factor, commonly denoted as $|K|^2$:

$$|K|^2 = \left| \frac{m^2 - 1}{m^2 + 2} \right|^2$$

In radar meteorology, $|K|^2$ depends heavily on the phase of the hydrometeor and the radar frequency. For microwave radar bands (like S-band or C-band NEXRAD), $|K|^2 \approx 0.93$ for liquid water droplets, while for solid ice particles, $|K|^2 \approx 0.176$.

When integrated over a unit volume containing a drop size distribution $N(D)$, this backscatter equation yields the radar volume reflectivity (η), which directly scales with the radar reflectivity factor $Z = \int D^6 N(D) dD$. This D^6 dependence is why the radar echo is overwhelmingly dominated by the largest drops in the Rayleigh regime.

To explore further:

Review the shift to the Mie regime for large hail

Examine non-spherical backscatter for dual-pol